Chapter-3

MATRIX

Matrix of Identity :-

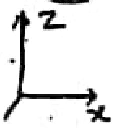
$$\begin{vmatrix} x \\ y \\ z \end{vmatrix} \xrightarrow{E} \begin{vmatrix} x,0,0 \\ 0,y,0 \\ 0,0,z \end{vmatrix} \equiv \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \qquad \boxed{\chi_E = +3}$$

$\boxed{\text{Character of Identity} = +3}$

Sum of diagonals of matrix from left to right is defined as character of matrix.

Matrix of Axis :-

C_2



$$\begin{vmatrix} x \\ y \\ z \end{vmatrix} \xrightarrow{C_{2z}} \begin{vmatrix} -x \\ -y \\ z \end{vmatrix} \equiv \begin{vmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \qquad \boxed{\chi_{C_2} = -1}$$

$$\begin{vmatrix} x \\ y \\ z \end{vmatrix} \xrightarrow{C_{2x}} \begin{vmatrix} x \\ -y \\ -z \end{vmatrix} \equiv \begin{vmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{vmatrix} \qquad \boxed{\chi_{C_2} = -1}$$

$$\begin{vmatrix} x \\ y \\ z \end{vmatrix} \xrightarrow{C_{2y}} \begin{vmatrix} -x \\ y \\ -z \end{vmatrix} \equiv \begin{vmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{vmatrix} \qquad \boxed{\chi_{C_2} = -1}$$

$\boxed{\text{Character of } C_2\text{-axis} = -1}$

C_4

$$\begin{vmatrix} x \\ y \\ z \end{vmatrix} \xrightarrow{C_{4z}} \begin{vmatrix} y \\ -x \\ z \end{vmatrix} \equiv \begin{vmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} \qquad \boxed{\chi_{C_4} = +1}$$

$$C_{nz} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \chi_{C_n} = 2\cos\theta + 1$$

$$C_{ny} = \begin{bmatrix} \cos\theta & \cancel{\sin\theta} & 0 \\ 0 & 1 & 0 \\ -\sin\theta & \cancel{\cos\theta} & \cos\theta \end{bmatrix} ; \quad C_{nx} = \cancel{\begin{bmatrix} 1 & 0 & 0 \\ \cos\theta & \sin\theta & 0 \\ -\cos\theta & \sin\theta & \end{bmatrix}}$$

$$C_{nx} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}$$

$\boxed{\text{Character of } C_\phi \text{ axis} = +1}$

C_3

C_3 : $\theta = 120^\circ$ $\cos 120^\circ = -\frac{1}{2}$, $\sin 120^\circ = \frac{\sqrt{3}}{2}$

$$C_{3z} = \begin{bmatrix} \cos 120^\circ & \sin 120^\circ & 0 \\ -\sin 120^\circ & \cos 120^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{\chi_{C_{3z}} = 0}$$

$\boxed{\text{Character of } C_3\text{-axis} = 0}$

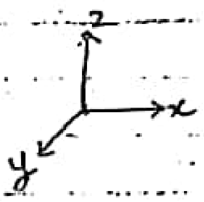
⇒ **Matrix of Improper Axis :-**

$$S_{nz} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & -1 \end{bmatrix}, \quad S_{ny} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ 0 & -1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$S_{nx} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}$$

$$\boxed{\chi_{S_n} = 2\cos\theta - 1}$$

⇒ **Matrix of Plane :-**



$$\begin{vmatrix} x \\ y \\ z \end{vmatrix} \xrightarrow{\sigma_{xz}} \begin{vmatrix} x \\ -y \\ z \end{vmatrix} \equiv \begin{vmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\boxed{\chi_{\sigma_{xz}} = 1}$$

$$\star \quad \begin{vmatrix} x \\ y \\ z \end{vmatrix} \xrightarrow{\sigma_{xy}} \begin{vmatrix} x \\ y \\ -z \end{vmatrix} \equiv \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{vmatrix}$$

$$\boxed{\chi_{\sigma} = +1}$$

$$\star \quad \begin{vmatrix} x \\ y \\ z \end{vmatrix} \xrightarrow{\sigma_{yz}} \begin{vmatrix} -x \\ y \\ z \end{vmatrix} \equiv \begin{vmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\boxed{\chi_{\sigma} = +1}$$

$\boxed{\text{Character of Plane} = +1}$

⇒ Matrix of Inversion ($i = S_2$) :-

$$S_{2z} = C_{2z} \cdot \sigma_{xy}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \xrightarrow{C_{2z}} \begin{bmatrix} -x \\ -y \\ z \end{bmatrix} \xrightarrow{\sigma_{xy}} \begin{bmatrix} -x \\ -y \\ -z \end{bmatrix} \equiv \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$\boxed{\text{Character of } S_{2z} \ (\chi_S) = -3}$

Gate 2007

Q.o:- 17. The S

Symmetry Element	Character
E	+3
$\sigma (\sigma_h, \sigma_d, \sigma_v)$	+1
C_2	−1
C_3	0
C_4	+1
S_2	+3
S_3	−2
S_4	−1

C_2, C_3, C_4: $2\cos\theta + 1$

S_2, S_3, S_4: $2\cos\theta - 1$

Scanned by CamScanner

Gate-2007

Qo-17. S_2 operation on a molecule with axis of rotation as z-axis moves a nucleus at (x, y, z) to

(a) $(-x, -y, +z)$ (b) $(x, -y, -z)$ (c) $(-x, y, -z)$ (d) $(-x, -y, -z)$.

Ans:- (d) $(-x, -y, -z)$

NET-June-2015

Qo-59. The product $C_{2x} \cdot \sigma_{xy}$ (C_{2x} is 2-fold rotation axis around x-axis and σ_{xy} is σxy mirror plane) ——

✓(a) σ_{xz} (b) σ_{yz} (c) C_{2y} (d) C_{2z}

Ans. (59) (a). σ_{xz}

$$\underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix} \xrightarrow{C_{2x}} \begin{bmatrix} x \\ -y \\ -z \end{bmatrix} \xrightarrow{\sigma_{xy}} \begin{bmatrix} x \\ -y \\ z \end{bmatrix}}_{\sigma_{xz}} \equiv \sigma_{xz}$$

NET-Dec-2014

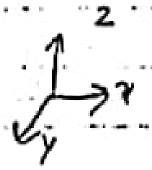
Qo:-139) The product $\sigma_{xy} \cdot S_{4z}$ (S_{4z} is 4-fold improper axi

✓(a) C_{4z} (b) $C_{4z} \cdot i$ (c) C_{4y} (d) C_{2z}.

~~Ans:- 139.)~~ ✓ $$\underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix} \xrightarrow{\sigma_{xy}} \begin{bmatrix} x \\ y \\ -z \end{bmatrix} \xrightarrow{C_{4z}} \begin{bmatrix} y \\ -x \\ -z \end{bmatrix} \xrightarrow{\sigma_{xy}} \begin{bmatrix} y \\ -x \\ z \end{bmatrix}}_{C_{4z}}$$

$$\sigma_{xy} \cdot S_{4z} = \sigma_{xy} \cdot C_{4z} \cdot \sigma_{xy}$$
$$= C_{4z} \cdot E$$

Ans. $= C_{4z}$



$$\begin{matrix} z \\ \\ \end{matrix} \xrightarrow{C_{2x}} \begin{matrix} z \\ -y \\ -z \end{matrix} \xrightarrow{C_{2y}} \begin{matrix} -z \\ -y \\ z \end{matrix}$$

NET-June-2014

Qo:- 143. The product of $C_{2x} \cdot C_{2y}$ is

(a) E (b) σ_{xy} ✓(c) C_{2z} (d) i

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \xrightarrow{C_{2x}} \begin{bmatrix} x \\ -y \\ -z \end{bmatrix} \xrightarrow{C_{2y}} \begin{bmatrix} -x \\ -y \\ z \end{bmatrix} = C_{2z}$$

$$\underbrace{\qquad\qquad\qquad}_{C_{2z}}$$

Qo: $C_{2x} \cdot C_{2y} : C_{2z}$ is

Ans. E.

Qo: $\sigma_{xy} \cdot \sigma_{xz} : \sigma_{yz}$ is

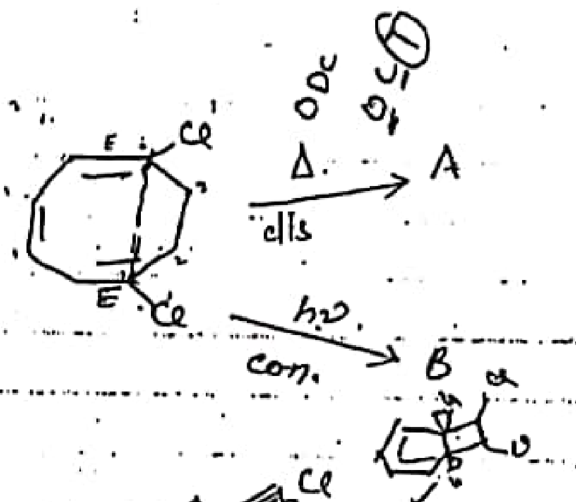
Ans.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \xrightarrow{\sigma_{xy}} \begin{bmatrix} x \\ y \\ -z \end{bmatrix} \xrightarrow{\sigma_{xz}} \begin{bmatrix} x \\ -y \\ -z \end{bmatrix} \xrightarrow{\sigma_{yz}} \begin{bmatrix} -x \\ -y \\ -z \end{bmatrix}$$

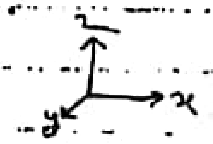
$$\underbrace{\qquad\qquad\qquad}_{i}$$

Inversion (i)

Qo:



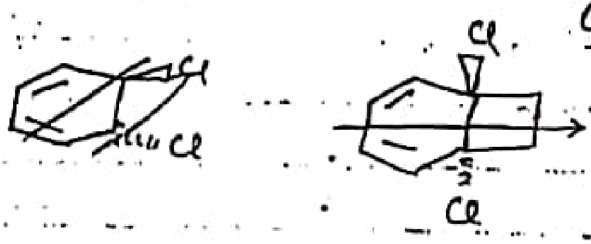
Product of plane in A & axis in B is



$\xrightarrow[\text{cis}]{\Delta}$

σ_{xy}

$\xrightarrow[\text{con.}]{h\nu}$



C_{2x}

$$\sigma_{xy} \cdot C_{2x} = \sigma_{xz}$$

$$\begin{vmatrix} x \\ y \\ z \end{vmatrix} \xrightarrow{\sigma_{xy}} \begin{vmatrix} x \\ y \\ -z \end{vmatrix} \xrightarrow{C_{2x}} \begin{vmatrix} x \\ -y \\ z \end{vmatrix}$$

$$\underbrace{\qquad\qquad\qquad}_{\sigma_{xz}}$$